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A THEORETICAL STUDY OF STAGNATION-POINT ABLATION

By LEONARD ROBERTS

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SUMMARY

A simplified analysis is made of the shielding mechanism which reduces the stagnation-point heat transfer when ablation takes place at the surface. The most significant result of the analysis is that the effective heat capacity of the ablation material increases linearly with stream enthalpy. The automatic shielding mechanism is discussed and the significant thermal properties of a good ablation material are given. The results of the analysis are given in terms of dimensionless parameters.

INTRODUCTION

It is well-known that the rate of heat transfer to a body is reduced when the nose or leading edges are blunt. Such a shape causes a thick boundary layer so that a shield of air over the nose or leading edge reduces heat transfer to the body. This reduction in heat transfer is not always sufficient to achieve desirable body temperatures, however, and further shielding is then required.

This shielding is accomplished by providing a material which in some way absorbs heat; for example, a copper nose of sufficient thickness absorbs enough heat to shield the interior of the body. Alternatively, the material may be supplied as a gas which is injected through a porous wall into the laminar boundary layer near the stagnation point. (See ref. 1.) Another method is to allow ablation to take place. This method is not just a negative approach in which heat is expended on the sublimation of unimportant material; it is also an extension of the blunt-nose idea of using the thickened boundary layer to shield the nose. The convection of heat by the foreign gas introduced can make the main contribution to the shielding at sufficiently high stagnation temperatures.

Previous theoretical studies of stagnation-point ablation have been concerned mainly with melting surfaces in which a layer of liquid flows over the surface (refs. 2 and 3); the main conclusion was that the convective shielding by the liquid layer is limited by the temperature at which the liquid boils. When vaporization occurs, however, the convective shielding in the gas boundary layer can increase considerably. Recently, a comprehensive review and study of the convective heat transfer with mass addition has been made (ref. 4) which includes the effects of dissociation, recombination, and combustion in the boundary layer. A reference list on the subject may be found in that report.

The purpose of the present report is to show that the mechanism of shielding by vaporization may be explained quantitatively and qualitatively by a simple analysis in which the more important physical aspects of the problem are correctly represented. The results of this analysis indicate the significant material properties that are required to make a good ablation shield.

SYMBOLS

- x coordinate along wall
- y coordinate normal to wall
- z transformed y-coordinate
- Z arbitrary value of z outside boundary layer
- u component of velocity in x-direction
- v component of velocity in y-direction

[1] Supersedes NACA Technical Note 4392 by Leonard Roberts, 1958.

U free-stream velocity in x-direction
C constant in velocity distribution
T temperature
$\overline{T}$ effective mean temperature (eq. (40))
W mass concentration of foreign gas
$\overline{W}$ effective mean concentration (eq. (38))
ρ density of mixture
μ coefficient of viscosity
k thermal conductivity
D_{12} coefficient of binary diffusivity
$\dot{m}$ rate of mass loss per unit area of wall
L latent heat of sublimation
S enthalpy ratio, $\frac{c_{p,2}(T_e-T_w)}{c_{p,2}(T_e-T_w)+L+c_b(T_w-T_b)}$
c_b specific heat of solid
c_p specific heat at constant pressure
$\overline{c}_p$ effective mean specific heat (eq. (43))
N_{Nu} Nusselt number
R_w Reynolds number
N_{Sc} Schmidt number
N_{Pr} Prandtl number
q heat-transfer rate per unit area
δ_u velocity boundary-layer thickness
δ_T thermal boundary-layer thickness
δ_W concentration boundary-layer thickness
θ thermal-layer thickness within the nose
K constant defined by equation (39)
H_{eff} effective heat capacity of ablation material

Subscripts:
e external flow, near stagnation point
w wall
b body
0 no injection
1 foreign gas
2 air

SHIELDING MECHANISM

The configuration considered is that shown in figure 1. The coordinate system is fixed in the surface at the stagnation point; thus, the interior of the body moves in the y-direction toward the surface with velocity v_b, the ablation rate. Within the solid in the steady state there is a balance of conductive and convective heat transfer.

At the surface ablation takes place and gases are produced; these gases are transported away from the wall and are convected in the laminar boundary layer near the stagnation point. This convection of gaseous material provides additional shielding and reduces the heat transfer to the surface. The rate of mass transfer at the surface is itself determined by the rate of heat transfer to the surface; thus this process is an automatic shielding mechanism. Additional heat transfer to the surface causes additional mass transfer with an attendant increase in shielding.

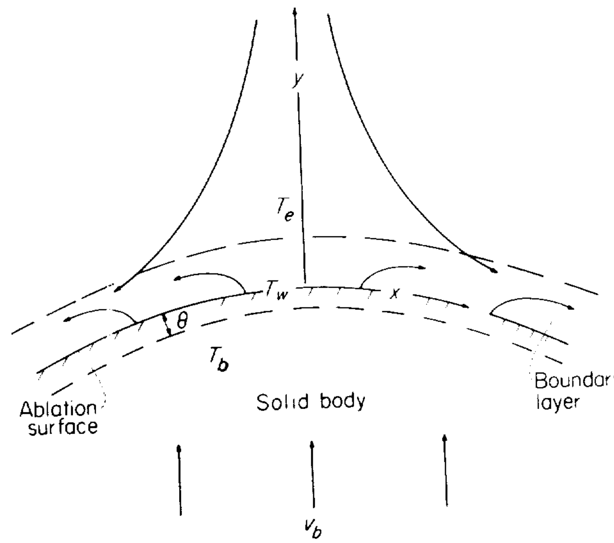


FIGURE 1.—Flow configuration for steady ablation.

In the steady state the boundary layer will consist of a velocity, temperature, and concentration boundary layer, each generally of different constant thickness. It is assumed that the component of velocity parallel to the wall u is linear in x and that the normal component of velocity v, the temperature T, the concentration of foreign gas W, and the properties of the mixture are functions only of y, the distance normal to the wall. (These assumptions result directly from the similarity nature of the flow and the absence of thermal and concentration gradients along the wall.)

For a given external flow and stagnation temperature the gas given off in the resulting ablation diffuses through the boundary layer and is convected with the air, as a mixture, under the action of the pressure gradient imposed by the external flow and the shear stress due to the presence of the wall. The rate of mass loss and the concentration of foreign gas at the wall are uniquely determined by the external-flow conditions and the properties of the solid material.

The overall solution of the problem is obtained by solving separately the heat-transfer problem in the solid and the heat and mass-transfer problem in the boundary layer; these solutions are then matched by the boundary conditions at the ablation surface.

ANALYSIS

HEAT TRANSFER IN THE SOLID

The transfer of heat in the solid moving with constant velocity v_b toward the surface is governed by the following equation:

$$\underset{\substack{\text{Diffusion}\\\text{of heat to}\\\text{interior}}}{\frac{d}{dy}\left(k_b\frac{dT}{dy}\right)} = \underset{\substack{\text{Convection}\\\text{of heat}\\\text{toward surface}}}{\dot{m}c_b\frac{dT}{dy}} \tag{1}$$

It is assumed that temperature gradients parallel to the surface are negligible and that c_b and k_b are constants. Equation (1) holds when the thickness of the solid is large compared with the thickness θ of the thermal layer (within the solid and near the surface) in which the temperature changes rapidly from its value at the wall T_w (the ablation temperature) to its value at the far interior T_b.

The solution of equation (1) which satisfies the conditions at $y=0$, $T=T_w$ (constant) and as $y\rightarrow-\infty$, $T=T_b$ (constant) is

$$T=T_b+(T_w-T_b)e^{\frac{\dot{m}c_by}{k_b}} \tag{2}$$

A characteristic thickness θ is defined by

$$\theta=\int_{-\infty}^{0}\frac{T-T_b}{T_w-T_b}dy=\frac{k_b}{\dot{m}c_b} \tag{3}$$

Thus, the heat content of the body per unit surface area is $c_b\theta(T_w-T_b)$. Consequently, in order to confine the high temperatures to a thin layer near the surface, the material should have low conductivity k_b and high specific heat c_b. Equation (3) also shows that the thermal layer decreases in thickness as the rate of mass loss increases.

From equation (2) the rate of transfer of heat away from the surface at $y=0$ is

$$\left(k_b\frac{dT}{dy}\right)_{w^-}=c_b(T_w-T_b)\dot{m} \tag{4}$$

where w^- denotes that the surface is approached through negative values of y. Equation (4) is a statement that the heat transfer rate at $y=0$ is that required to raise the mass $\dot{m}$ of specific heat c_b through the temperature difference T_w-T_b. The rate of mass loss $\dot{m}$ is unknown at this stage of the analysis and must be determined by consideration of the boundary-layer flow over the surface of the wall.

WALL CONDITIONS FOR ABLATION

The heat-transfer condition at the wall is

$$\underset{\substack{\text{Heat transfer}\\\text{to wall}}}{\left(k\frac{dT}{dy}\right)_w} = \underset{\substack{\text{Latent}\\\text{heat}}}{L\dot{m}} + \underset{\text{Body heat}}{c_b(T_w-T_b)\dot{m}} \tag{5}$$

The last term of equation (5) is obtained from equation (4) and is the heat lost when the temperature of the mass $\dot{m}$ is raised to temperature T_w before ablation.

The mass-transfer condition is

$$\underset{\substack{\text{Diffusion of}\\\text{gas away}\\\text{from wall}}}{-\rho_wD_{12}\frac{dW}{dy}} + \underset{\substack{\text{Convection of}\\\text{gas away}\\\text{from wall}}}{\rho_wv_wW} = \underset{\substack{\text{Gas introduced}\\\text{at wall}}}{\dot{m}} \tag{6}$$

where the subscript w refers to conditions that exist as the wall is approached through positive values of y.

A third condition is that which states that there is no net transfer of air into the wall:

$$\underset{\substack{\text{Diffusion of air}\\\text{toward wall}}}{-\left(\rho D_{12}\frac{dW}{dy}\right)_w} = \underset{\substack{\text{Convection of air}\\\text{away from wall}}}{(1-W_w)\rho_wv_w} \tag{7}$$

that is, there is a balance of diffusion of air toward the wall because of the concentration gradient and convection of air, of concentration $1-W_w$, away from the wall. Elimination of the term $\left(\rho_wD_{12}\frac{dW}{dy}\right)_w$ from equations (6) and (7) gives $\rho_wv_w=\dot{m}$ which may be taken to be a definition of v_w. The conditions denoted by equations (5), (6), and (7) are required for the solution of the boundary-layer equations.

BOUNDARY-LAYER INTEGRAL EQUATIONS

The method used in this section to determine the effect on the heat transfer of convective shielding in the boundary layer is that formulated in reference 1. The principal equations are repeated herein (with some modification) for the sake of completeness.

The boundary-layer equations for two-dimensional flow may be written as follows:

$$-\rho_w V(Z) \quad + \quad \dot{m} \quad = \quad C\rho_w \int_0^Z \frac{u}{U} dz \tag{8}$$

Mass flow into boundary layer from external stream; Mass flow at wall; Convection of mass along wall

$$\dot{m} \quad = \quad C\rho_w \int_0^Z W \frac{u}{U} dz \tag{9}$$

Gas introduced; Gas convected

$$-c_{p,2}(T_e - T_w)\rho_w V(Z) \quad - \quad \left(k\frac{dT}{dy}\right)_w \quad = \quad C\rho_w \int_0^Z c_p(T - T_w)\frac{u}{U} dz \tag{10}$$

Heat flow into boundary layer from external stream; Heat transfer to wall; Convection of heat in boundary layer along wall

In equations (8), (9), and (10) C is the constant which appears in the external-velocity distribution

$$U = Cx$$

$$V(Z) = \frac{\rho v}{\rho_w}$$

$$z = \int_0^y \frac{\rho}{\rho_w} dy$$

and Z is some arbitrary value of z outside the boundary layer.

The quantity $V(Z)$ may be eliminated from equation (10) by use of the relation

$$c_p = c_{p,1} W + c_{p,2}(1 - W) \tag{11}$$

and equations (8) and (9); the result is written

$$C\rho_w \int_0^Z c_p(T_e - T) \frac{u}{U} dz = \left(k\frac{dT}{dy}\right)_w + c_{p,1}(T_e - T_w)\dot{m} \tag{12}$$

Alternatively, by use of the boundary condition of equation (5), equation (12) has the form

$$C\rho_w \int_0^Z c_p(T_e - T) \frac{u}{U} dz = [c_{p,1}(T_e - T_w) + L + c_b(T_w - T_b)]\dot{m} \tag{13}$$

The equations for transfer of heat and mass near the three-dimensional stagnation point also reduce to equations (12) and (13) if x is measured from the stagnation point and $z = 2\int_0^y \frac{\rho}{\rho_w} dy$.

HEAT-TRANSFER RELATIONS FOR NO ABLATION

The rate of mass loss experienced by the body is determined in part by the aerodynamic heat transfer which would prevail in the absence of ablation; this rate is denoted by q_0.

The heat-transfer relations here are found in terms of the Nusselt number and Reynolds number which have the usual definitions near the stagnation point:

$$N_{Nu,w} = \frac{x}{T_e - T_w}\left(\frac{dT}{dy}\right)_w$$

$$R_w = C\frac{\rho_w}{\mu_w} x^2$$

In addition, the Prandtl number and Schmidt number are defined as

$$N_{Pr,w} = \frac{\mu_w}{k_w} c_{p,2}$$

$$N_{Sc,w}=\frac{\mu_w}{\rho_w D_{12}}$$

The rate of transfer of heat q_0 to the wall, when there is no ablation, is then given by

$$q_0=\left(k\frac{dT}{dy}\right)_w$$
$$=c_{p,2}(T_e-T_w)(\rho_w\mu_w C)^{1/2}\frac{1}{N_{Pr,w}}\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0 \quad (14)$$

The dimensionless quantity $\frac{N_{Nu,w}}{R_w^{1/2}}$ used herein is that suggested in reference 5 and used in reference 1. For the axisymmetric flow,

$$\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0=0.765\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.4}N_{Pr,w}^{0.4} \quad (15)$$

and for the two-dimensional flow,

$$\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0=0.570\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.4}N_{Pr,w}^{0.4} \quad (16)$$

In order to obtain an approximate expression for the thickness [2] $\delta_{u,0}$ of the velocity boundary layer, equation (12) with $\dot{m}=0$ is written in the form

$$\frac{1}{\delta_{u,0}}\int_0^z\frac{T_e-T}{T_e-T_w}\frac{u}{U}dz=\frac{1}{\delta_{u,0}}\left(\frac{\mu_w}{\rho_w C}\right)^{1/2}\frac{1}{N_{Pr,w}}\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0 \quad (17)$$

and the linear profiles

$$\left.\begin{aligned}\frac{u}{U}&=\frac{z}{\delta_{u,0}}\\ \frac{T_e-T}{T_e-T_w}&=1-\frac{z}{\delta_{T,0}}\end{aligned}\right\} \quad (18)$$

are used in order to evaluate the integral in equation (17). (The quantity $\delta_{T,0}$ is the thickness of the thermal boundary layer.)

Equation (17) then has the approximate form (ref. 1):

$$\frac{1}{6}\left(\frac{\delta_T}{\delta_u}\right)_0^2=\frac{1}{\delta_{u,0}}\left(\frac{\mu_w}{\rho_w C}\right)^{1/2}\frac{1}{N_{Pr,w}}\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0 \quad (19)$$

Equations (15) and (16) show that the effect of Prandtl number on the heat-transfer parameter is given by

$$\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0=\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0^{(1)}N_{Pr,w}^{0.4} \quad (20)$$

where the superscript (1) denotes the value when $N_{Pr,w}=1$.

When $N_{Pr,w}=1$, the thermal and viscous diffusive effects are similar; thus, $\frac{\delta_T}{\delta_u}=1$. From equation (19),

$$\delta_{u,0}\left(\frac{\rho_w C}{\mu_w}\right)^{1/2}=6\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0^{(1)} \quad (21)$$

The dimensionless thickness $\delta_{u,0}\left(\frac{\rho_w C}{\mu_w}\right)^{1/2}$ is assumed to be independent of the Prandtl number; thus, when equations (20) and (21) are inserted into equation (19), the following result is obtained:

$$\left(\frac{\delta_T}{\delta_u}\right)_0=N_{Pr,w}^{-0.3} \quad (22)$$

In the following section it is assumed also that the relation

$$\frac{\delta_T}{\delta_u}=N_{Pr,w}^{-0.3}$$

is valid even when ablation takes place.

BOUNDARY-LAYER FLOW WITH ABLATION

Consider now the effect, on the boundary layer, of ablation at the wall. The boundary layer becomes thicker and the profiles may change (because of the increase in mass flow) with a resulting reduction of gradients in the boundary layer. In particular, the heat transfer to the wall is reduced since the heat convected parallel to the wall in the boundary layer is increased.

The increase in boundary-layer thickness is found by considering the mass flow in the boundary layer in the direction parallel to the wall:

$$\underset{\substack{\text{Mass flow}\\ \text{with ablation}}}{C\rho_w\int_0^{\delta_u}\frac{u}{U}dz} = \underset{\substack{\text{Mass flow with}\\ \text{no ablation}}}{C\rho_w\int_0^{\delta_{u,0}}\frac{u}{U}dz} + \underset{\substack{\text{Mass}\\ \text{ablated}}}{\dot{m}}$$

which gives

$$\frac{1}{2}\delta_u=\frac{1}{2}\delta_{u,0}+\frac{\dot{m}}{C\rho_w} \quad (23)$$

when the linear profiles $\frac{u}{U}=\frac{z}{\delta_{u,0}}$ and $\frac{u}{U}=\frac{z}{\delta_u}$ for no

[2] The quantity $\delta_{u,0}$ is more correctly related to twice the velocity-layer thickness in the axisymmetric case.

ablation and ablation, respectively, are used.

In order to keep the ideas simple, consider first the case in which the gas given off has properties identical with those of air. (The more general case will then be given as an extension of this method.)

The heat-transfer equation (eq. (12)) may be written

$$\frac{1}{\delta_u}\int_0^z \frac{T_e-T}{T_e-T_w}\frac{u}{U}\,dz = \frac{1}{\delta_u}\left(\frac{u_w}{\rho_w C}\right)^{1/2}\left[\frac{1}{N_{Pr,w}}\frac{N_{Nu,w}}{R_w^{1/2}}+\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}}\right]$$

by use of equation (14). Equations (18), (23), and (21) are used to simplify this equation with the following result:

$$\frac{1}{N_{Pr,w}}\frac{N_{Nu,w}}{R_w^{1/2}}=\left(\frac{1}{N_{Pr,w}}\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0-\left(1-\frac{1}{3}N_{Pr,w}^{-0.6}\right)\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}} \qquad (24)$$

where $\frac{1}{N_{Pr,w}}\frac{N_{Nu,w}}{R_w^{1/2}}$ is given by equation (15) or (16). Equation (24) rewritten in terms of the heat-transfer rate q_0 by using equations (14) and (15) and the relation

$$q=c_{p,2}(T_e-T_w)(\rho_w\mu_w C)^{1/2}\frac{1}{N_{Pr,w}}\frac{N_{Nu,w}}{R_w^{1/2}}$$

is

$$q_0=\Big[\underbrace{c_b(T_w-T_b)}_{\text{Body heat}}+\underbrace{L}_{\text{Latent heat}}+\underbrace{c_{p,2}(T_e-T_w)\left(1-\frac{1}{3}N_{Pr,w}^{-0.6}\right)}_{\text{Boundary-layer shielding}}\Big]\dot{m} \qquad (25)$$

The quantity $\left(1-\frac{1}{3}N_{Pr,w}^{-0.6}\right)(T_e-T_w)$ is the effective temperature rise in the boundary layer of the gas given off in ablation.

When q_0 is expressed in terms of dimensionless quantities by use of equations (14) and (20), equation (25) becomes

$$\frac{c_{p,2}(T_e-T_w)}{c_{p,2}(T_e-T_w)+L+c_b(T_w-T_b)}=\frac{N_{Pr,w}^{0.6}\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}}}{\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0^{(1)}+\frac{1}{3}\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}}} \qquad (26)$$

Equation (26) may be used to determine $\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}}$ when the enthalpy ratio

$$S=\frac{c_{p,2}(T_e-T_w)}{c_{p,2}(T_e-T_w)+L+c_b(T_w-T_b)}$$

is known. The result is

$$\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}}=\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0^{(1)}\frac{N_{Pr,w}^{-0.6}S}{1-\frac{1}{3}N_{Pr,w}^{-0.6}S} \qquad (27)$$

As a check on the validity of the several simplifying assumptions that have been made, a comparison is made with exact solutions for air-to-air injection which were compared with the present method in reference 1 for axisymmetric flow. When ablation occurs, the amount of gas introduced into the boundary layer is no longer a free parameter, as in the case of injection, but is determined by the rate of heat transfer according to the following equation:

$$\frac{1}{N_{Pr,w}}\frac{N_{Nu,w}}{R_w^{1/2}}=\frac{L+c_b(T_w-T_b)}{c_{p,2}(T_e-T_w)}\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}} \qquad (28)$$

which is the dimensionless form of equation (5).

Thus, the dimensionless rate of mass loss $\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}}$ is determined by the intersection of the straight line

$$\frac{N_{Nu,w}}{R_w^{1/2}}=N_{Pr,w}\frac{1-S}{S}\frac{\dot{m}}{(\rho_w\mu_w C)^{1/2}}$$

(which is eq. (28) by definition of S) with the straight line given by equation (24). This intersection has been determined for various values of S and the results are shown in figure 2.

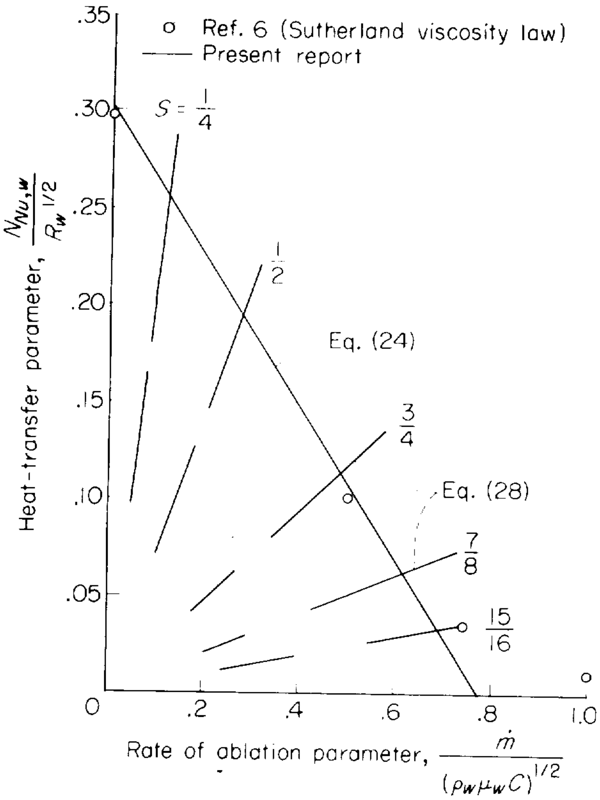


FIGURE 2.—Rate of ablation when foreign gas is same as air. Two-dimensional; $N_{Pr,w}=0.7$; $\frac{T_w}{T_e}=0.05$.

The agreement between the results of the present analysis and the exact air-to-air injection results of reference 6 is extremely good except for values of the parameter S very near unity. The latter situation occurs when the latent heat L and the body heat $c_b(T_w-T_b)$ are small. Such a situation is undesirable; thus this range of S is unimportant from a practical viewpoint. The foregoing comparison is considered to be sufficient justification to extend the method to take account of the additional effects which occur when the gas has properties different from those of air.

The mechanism by which air diffuses through the boundary layer in a binary mixture is exactly analogous to that by which heat diffuses in the absence of viscous dissipation when $c_{p,1}=c_{p,2}$. The analogy is seen when the gas-transfer equation (9) is written

$$C\rho_w\int_0^z \frac{W}{W_w}\frac{u}{U}\,dz=\frac{\dot m}{W_w} \tag{29}$$

and compared with the heat-transfer equation when $c_{p,1}=c_{p,2}$

$$C\rho_w\int_0^z \frac{T_e-T}{T_e-T_w}\frac{u}{U}\,dz=\frac{\dot m}{S} \tag{30}$$

obtained by the use of equation (13) and the definition of S.

The boundary condition at the wall which is obtained from equation (7)

$$-\left[\rho D_{12}\frac{d}{dy}\left(\frac{W}{W_w}\right)\right]_w=\frac{1-W_w}{W_w}\dot m \tag{31}$$

is compared with

$$-\left[\rho\frac{k}{\rho c_{p,2}}\frac{d}{dy}\left(\frac{T_e-T}{T_e-T_w}\right)\right]_w=\frac{1-S}{S}\dot m \tag{32}$$

which is obtained from equation (5) and the definition of S. The analogous quantities are given in the following table:

Binary diffusion	Thermal diffusion
W	$\frac{c_{p,2}(T_e-T)}{c_b(T_w-T_b)+L+c_{p,2}(T_e-T_w)}$
D_{12}	$\frac{k}{\rho c_{p,2}}$
$N_{Sc}=\frac{\mu}{\rho D_{12}}$	$N_{Pr}=\frac{\mu}{k}c_{p,2}$
W_w	$\frac{c_{p,2}(T_e-T_w)}{c_b(T_w-T_b)+L+c_{p,2}(T_e-T_w)}=S$

Because of this analogy the following relations may be written:
From equation (22),

$$\frac{\delta_W}{\delta_u}=N_{Sc,w}^{-0.3} \tag{33}$$

where δ_W is the thickness of the concentration boundary layer, and from equation (26)

$$W_w=\frac{N_{Sc,w}^{0.6}\,\frac{\dot m}{(\rho_w\mu_w C)^{1/2}}}{\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0^{(1)}+\frac{1}{3}\,\frac{\dot m}{(\rho_w\mu_w C)^{1/2}}} \tag{34}$$

where $\left(\frac{N_{Nu,w}}{R_w^{1/2}}\right)_0^{(1)}$ is given by equation (15) or (16).

Equation (34) gives the concentration of foreign gas at the wall, which is required for the determination of the boundary-layer shielding (since the specific heat of the mixture depends on W). The heat-transfer equation (12) is rewritten by using equations (5) and (11) as

$$C\rho_w\int_0^Z [c_{p,2}+(c_{p,1}-c_{p,2})W](T_e-T)\frac{u}{U}\,dz=[c_{p,1}(T_e-T_w)+L+c_b(T_w-T_b)]\dot{m} \tag{35}$$

and the linear profile

$$\frac{W}{W_w}=1-\frac{z}{\delta_W} \tag{36}$$

is assumed.

Substitution of equations (18) and (36) into equation (35) gives an equation analogous to equation (25) but includes the effect of the difference in specific heats and a binary diffusion (in the quantity $N_{Sc,w}$). This result is written in the form (see ref. 1 for a more complete derivation):

$$q_0=\Big\{\underbrace{c_b(T_w-T_b)}_{\text{Body heat}}+\underbrace{L}_{\text{Latent heat}}+\underbrace{[c_{p,1}\overline{W}+c_{p,2}(1-\overline{W})]\left(1-\frac{1}{3}N_{Pr,w}^{-0.6}\right)(T_e-T_w)}_{\text{Boundary-layer shielding}}\Big\}\dot{m} \tag{37}$$

where $\overline{W}$ is an effective concentration of foreign gas in the boundary layer and is defined by

$$\overline{W}=\frac{1-KN_{Sc,w}^{0.6}}{1-\frac{1}{3}N_{Pr,w}^{-0.6}} \tag{38}$$

where

$$K=6\frac{1}{\delta_u}\int_0^Z\frac{W}{W_w}\frac{T_e-T}{T_e-T_w}\frac{u}{U}\,dz \tag{39}$$

Again the effective temperature rise of the convected gas in the boundary layer is $\left(1-\frac{1}{3}N_{Pr,w}^{-0.6}\right)(T_e-T_w)$ and the effective temperature of the convected gas is

$$\overline{T}=T_w+\left(1-\frac{1}{3}N_{Pr,w}^{-0.6}\right)(T_e-T_w) \tag{40}$$

which increases with $N_{Pr,w}$. (See fig. 3.)

When q_0 is expressed in terms of the enthalpy difference across the boundary layer by use of equations (14) and (15) or (16), the following expressions are obtained for the rate of mass loss:

For axisymmetric flow,

$$\dot{m}=\frac{0.765\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.4}N_{Pr,w}^{-0.6}(\rho_w\mu_wC)^{1/2}c_{p,2}(T_e-T_w)}{c_b(T_w-T_b)+L+\overline{c}_p(\overline{T}-T_w)} \tag{41}$$

and for two-dimensional flow,

$$\dot{m}=\frac{0.570\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.4}N_{Pr,w}^{-0.6}(\rho_w\mu_wC)^{1/2}c_{p,2}(T_e-T_w)}{c_b(T_w-T_b)+L+\overline{c}_p(\overline{T}-T_w)} \tag{42}$$

where

$$\overline{c}_p=c_{p,1}\overline{W}+c_{p,2}(1-\overline{W}) \tag{43}$$

is the effective specific heat.

Equations (40) and (41) show that

$$\dot{m}\rightarrow\frac{0.765\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.4}(\rho_w\mu_wC)^{1/2}N_{Pr,w}^{-0.6}}{\frac{\overline{c}_p}{c_{p,2}}\left(1-\frac{1}{3}N_{Pr,w}^{-0.6}\right)}$$

for large values of T_e-T_w. The effective total heat capacity of the shielding material is given by equation (37) as

$$H_{eff}=\frac{q_0}{\dot{m}}=c_b(T_w-T_b)+L+\overline{c}_p(\overline{T}-T_w) \tag{44}$$

When the expression for $\overline{T}$ given by equation (40) is inserted into equation (44) it is seen that H_{eff} is a linear function of T_e-T_w having intercept

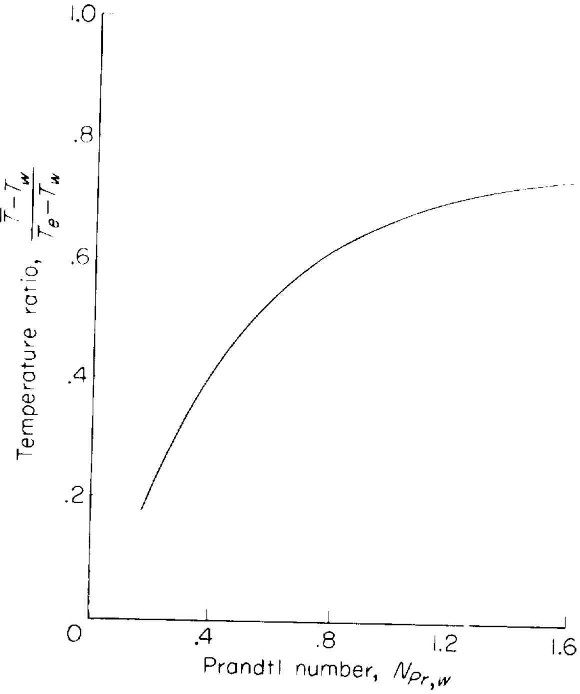


FIGURE 3.—Effect of $N_{Pr,w}$ on $\overline{T}$.

equal to $c_b(T_w-T_b)+L$ and slope $\overline{c}_p\left(1-\frac{1}{3}N_{Pr,w}{}^{0.6}\right)$, that is

$$H_{eff}=c_b(T_w-T_b)+L+\overline{c}_p\left(1-\frac{1}{3}N_{Pr,w}{}^{0.6}\right)(T_e-T_w)$$

DISCUSSION

It is seen from equations (41) and(40) that the rate of mass loss is reduced by choosing a material such that the enthalpy ratio

$$\frac{c_b(T_w-T_b)+L+\overline{c}_p(T_e-T_w)\left(1-\frac{1}{3}N_{Pr,w}{}^{-0.6}\right)}{c_{p,2}(T_e-T_w)N_{Pr,w}{}^{-0.6}}$$

is as large as possible; this condition requires that the following quantities be large:

(a) The surface ablation temperature T_w
(b) The specific heat of the solid c_b
(c) The latent heat L
(d) The effective specific heat $\overline{c}_p$
(e) The Prandtl number $N_{Pr,w}$

In addition, as shown in equation (3), the thermal conductivity k_b of the solid material should be small in order to confine the high surface temperature to a thin thermal layer near the surface.

The effective specific heat $\overline{c}_p$ is large when $c_{p,1}>c_{p,2}$ and $\overline{W}$ is large. Since $\overline{W}$ increases as $N_{Sc,w}$ decreases and is virtually independent of $N_{Pr,w}$ for $0.6<N_{Pr,w}<1.6$ (fig. 4), $N_{Sc,w}$ should be as small as possible. When $c_{p,1}<c_{p,2}$, however, $\overline{W}$ should be small and hence $N_{Sc,w}$ should be large in order to reduce the rate of mass loss. These diffusive effects on $\overline{c}_p$ are explained physically in reference 1 and may be summarized briefly as follows: When $c_{p,1}>c_{p,2}$ and $N_{Sc,w}$ is small, the foreign gas diffuses quickly through the boundary layer before being convected; when $c_{p,1}<c_{p,2}$ and $N_{Sc,w}$ is large, the gas remains near the wall but displaces the air away from the wall. In both cases the fluid of greater specific heat is convected in that part of the flow at the highest temperature and with the highest velocity, and thus produces the most convective shielding. (See fig. 5.)

In order to bring out the more important shielding parameters $\left(\frac{c_{p,1}}{c_{p,2}}\text{ and }S\right)$ without reference to any particular material, the Prandtl number and Schmidt number are given the value of 1; whereas in a more general calculation they would be determined by the concentration W and the properties of the material in the gaseous state. (See ref. 1.) A further simplification is brought about by the approximation

$$\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0.4}(\rho_w\mu_w C')^{1/2}\approx(\rho_e\mu_e C')^{1/2}$$

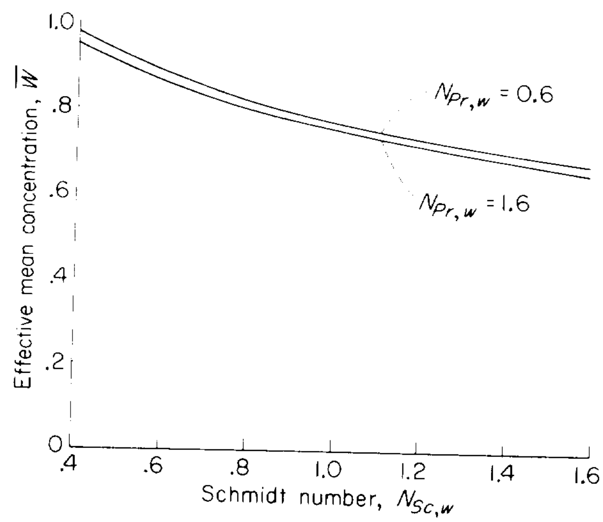
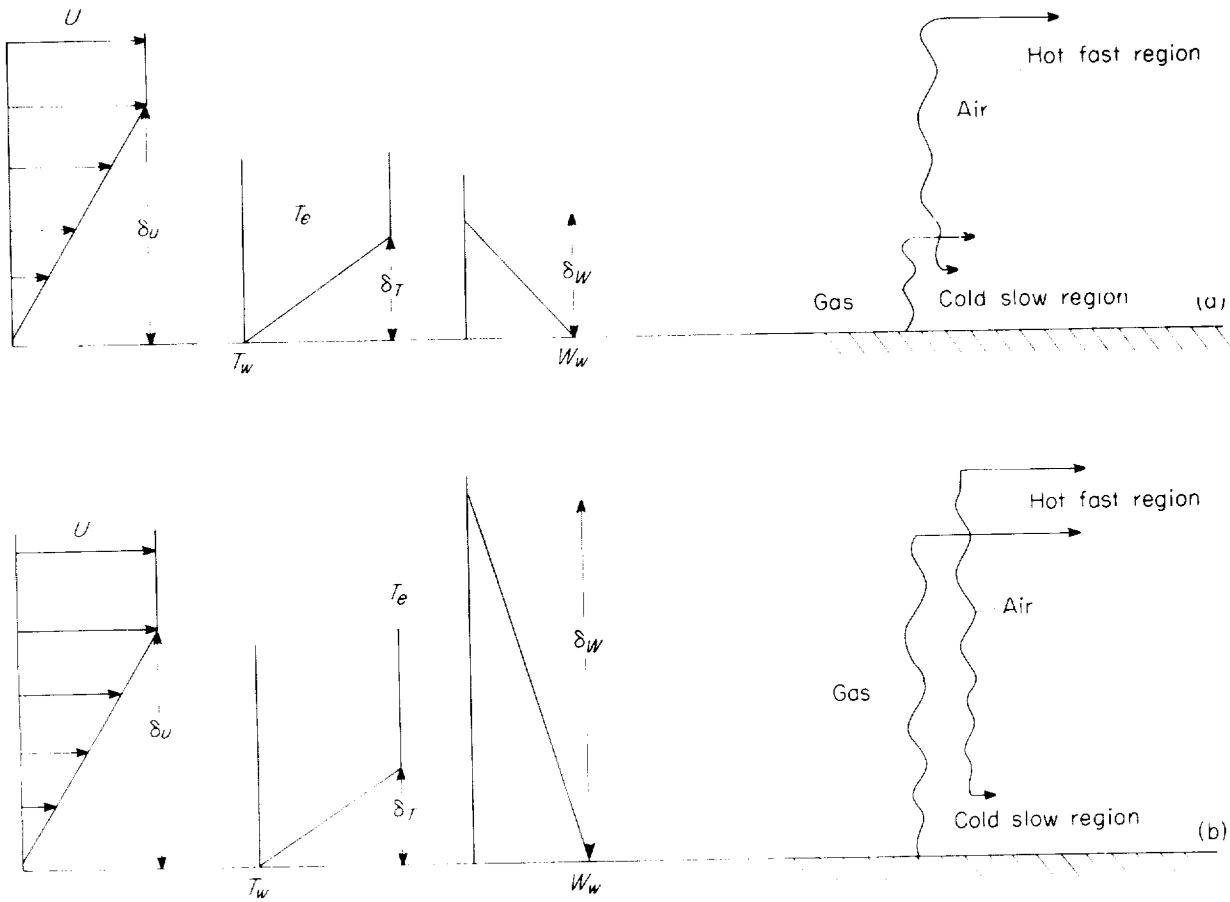


FIGURE 4.—Variation of $\overline{W}$ with $N_{Sc,w}$. ($\overline{W}$ is virtually independent of $N_{Pr,w}$.)



(a) $c_{p,2} > c_{p,1}$; maximum shielding when $N_{Pr,2}$ large and $N_{Sc,w}$ large.

(b) $c_{p,1} > c_{p,2}$; maximum shielding when $N_{Pr,2}$ large and $N_{Sc,w}$ small.

Figure 5. Schematic diagram of diffusive effects.

in equations (41) and (42). Figure 6 shows the rates of mass loss as a function of $\frac{c_{p,1}}{c_{p,2}}$ and S for axisymmetric and two-dimensional flow; these rates were determined by using these assumptions.

CONCLUDING REMARKS

The cooling of a blunt body by ablation appears to be a promising method since the shielding mechanism is such that gas shield increases as the enthalpy of the stream increases. Furthermore, the effective heat capacity of material increases even though the rate of mass loss becomes almost constant as the heating rate increases. The most important thermal properties for a good ablation material are: low thermal conductivity, high specific heat of solid, and high specific heat in the gaseous state. The simple quantitative results of the analysis bear out the physical ideas of the ablation mechanism and agree well with the results of an exact analysis for an air-to-air injection.

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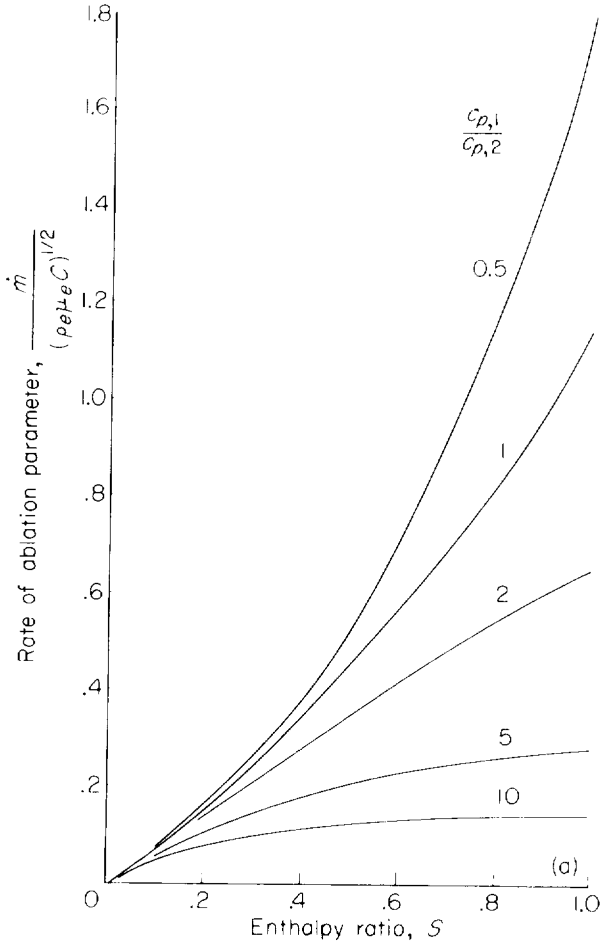
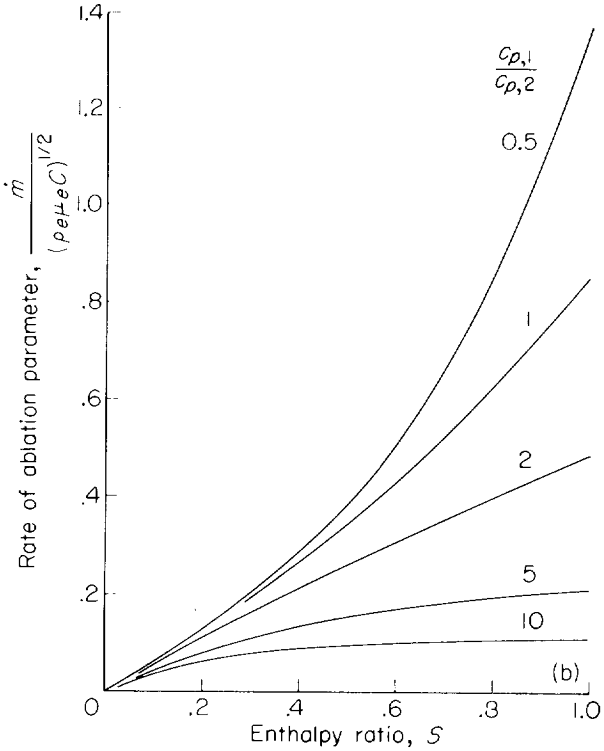


(a) Axisymmetric flow; $N_{Pr,w} = N_{Sc,w} = 1$.

FIGURE 6.—Rate of ablation as a function of $\frac{c_{p,1}}{c_{p,2}}$ and S.



(b) Two-dimensional flow; $N_{Pr,w} = N_{Sc,w} = 1$.

FIGURE 6.—Concluded.

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